Adjoint vs Inverse Migration Operator

Adjoint Image $m_{\text{adj}} = L^T d$

True Inverse Image $m_{\text{inv}} = (L^T L)^{-1} L^T d$

The missing factor $(L^T L)^{-1}$is where illumination and amplitude correction happen.

Consequences

- Bright spots where illumination is high.
- Dim or missing reflectors where illumination is poor.
- Directional bias (e.g., steep dips recovered better).
- Incorrect amplitudes (adjoint only produces a "reverse propagation" image, not reflectivity).

Adjoint migration is therefore not energy-balanced.

Adjoint migration

- Back projects data.
- Produces an image but with uneven sensitivity.
- Does not correct for acquisition geometry or wave propagation effects.
- Leads to illumination artifacts.

Inverse (least-squares) migration

- Solves the true mathematical inverse problem.
- Balances the image by normalizing with the illumination operator $L^T L$.
- Removes acquisition bias and illumination artifacts.
- Produces amplitude-correct reflectivity.

Difference:

- The adjoint operator simply back projects data, while the inverse operator solves the full inverse problem (often through least-squares) to recover the true reflectivity model.

Why illumination issues arise with the adjoint:

- Because L^Tdoes not compensate for uneven sensitivity in the forward operator L.
- True inversion requires correcting with $(L^T L)^{-1}$, which redistributes energy and removes illumination bias.
- Without this correction, adjoint migration overemphasizes well-illuminated areas and underrepresents poorly illuminated areas.