## Level-set inversions

Level-set inversions are a mathematical and computational tool used in inversions to determine the shape or boundary of a subsurface body.

**The core idea**:

• You represent the boundary of an object not by explicitly drawing its edges but by using a higher-dimensional function called a level-set function.

• The object is defined implicitly as all points where this function equals zero.

• During inversion, you adjust the level-set function to minimize the mismatch between observed data (gravity, magnetic or seismic) and predictions from a physical model.

• As the function evolves, its zero-level set morphs naturally into complex shapes, even splitting or merging if needed.

**Why is this great?**

• It handles topological changes gracefully

• It focuses the inversion effort on the geometry rather than detailed property values.

• It meshes nicely with regularization and prior information.

**The Scenario**

Suppose we're mapping a region where sedimentary rocks (light and non-magnetic) blanket an intrusive body (dense and magnetic). Gravity data suggests extra mass. Magnetic data that there's a body with strong magnetization. But we don't know its **shape**, only its effects.

**Level-Set Inversion**

We introduce a level-set function, let's call it $\phi(x)$. Wherever $\phi(x) > 0$, we assume intrusive rock with high density and high susceptibility. Wherever $\phi(x) < 0$, we assume the surrounding sediments with lower properties. The **zero contour** of $\phi(x)$ represents the unknown geological boundary.

We forward model gravity and magnetic fields based on the current $\phi$-defined geometry and compare them to our measured anomalies. If the mismatch (misfit) is high, our model is wrong. Gradient-based optimization nudges $\phi(x)$ to reduce the error, reshaping the boundary.

**Why This Is Powerful**

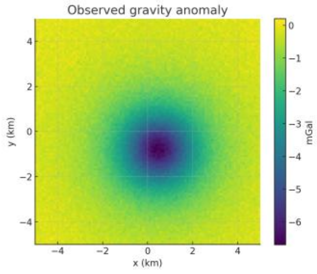
Traditional voxel-based inversions might smear dense/magnetic properties everywhere. Level-set inversion instead gives you a crisp, interpretable boundary.

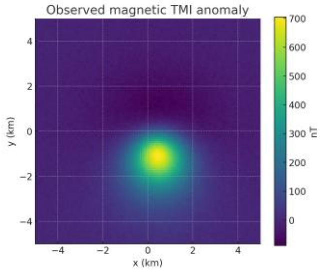
**Example**

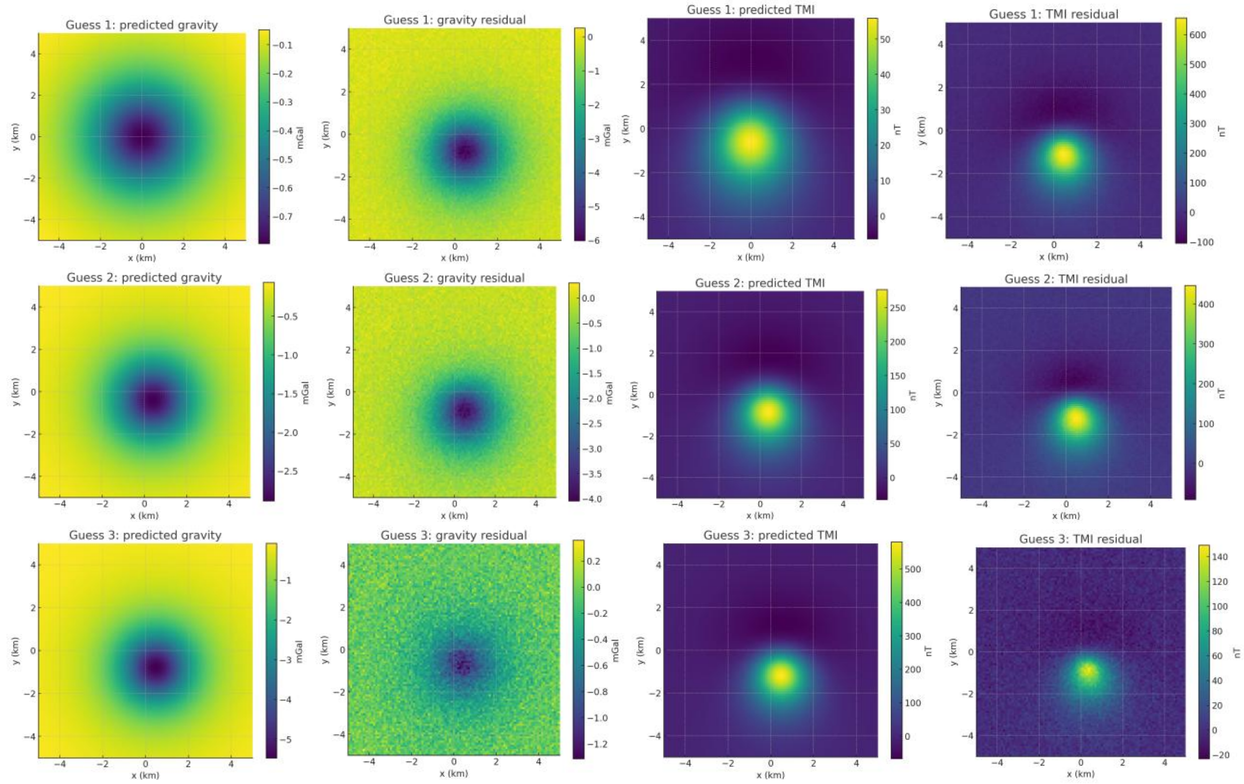
A joint gravity–magnetics synthetic example. We simulate a 10x10 km survey for a dense, magnetic intrusion (a dome + feeder dike via spheres), generate noisy gravity and magnetic "observed" data, then show successive boundary updates (coarse-to-fine guesses) to make the synthetic data fit the "observed" data.

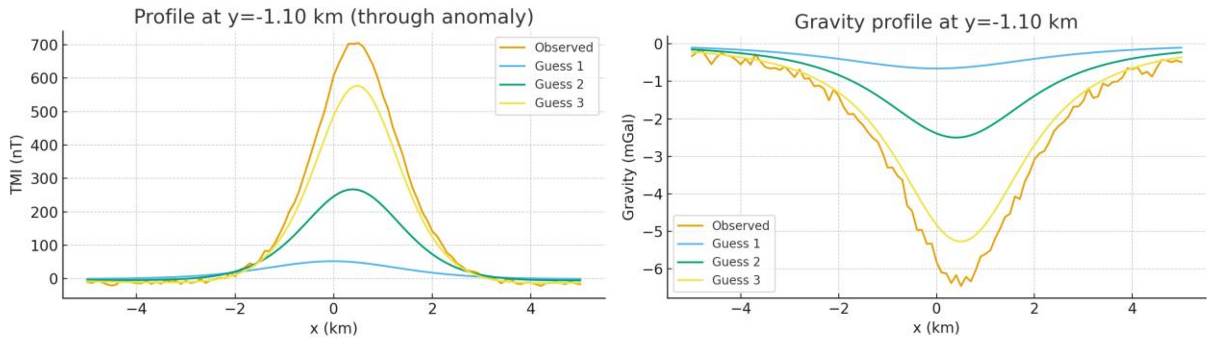
*Notes*:

- Gravity: use point-mass approximation for spheres (valid inside the body).
- Magnetics: TMI anomaly from induced dipoles, approximating each sphere as a dipole at its centre.
- Earth's field set to a realistic mid-latitude value for the Netherlands region.
- We add realistic noise: 0.1 mGal for gravity, 5 nT for magnetics.
- "Inversion" is a sequence of 3 updates to make the synthetic data fit the "observed" data.






In short, the density and susceptibility are assumed to be reasonably known, only the shape needs to be determined.