Gassmann (G) versus Brown & Korringa (B&K)

A discussion took place between Leon Thomsen and Yury Alkhimento in Geophysics 2024 , N2, whether Gassmann's equation can be used with confidence. Thomsen shows the difference between G and B&K is that B&K contains an additional parameter $\mathbf{K_M}$, which is the volume weighted mean compressibility that includes the compressibility of the solid and the pore spaces.
Thomsen states that Gassmann is only valid when $\mathbf{K_M}=\mathbf{K_S}$, which is valid for a specific rock configuration. Alkhimento claims that experiments have shown that the application of Gassmann is valid in general.
It should be noted that non-viscous wave propagation assumes undrained / closed system $\mathbf{K_{ud}}$, whereas, according to Thomsen, the error made in the derivation is that the logic of a drained/open system is used in the derivation of the Gassmann equation. It is not clear to me and maybe others, how much this error influences the use of Gassmann, especially in 4D.

Gassmann:

$$\mathbf{K}_{ud} = \mathbf{K}_{fr} + \frac{(1 - \mathbf{K}_{fr}\mathbf{K}_S^{-1})^2}{\phi(\mathbf{K}_F^{-1} - \mathbf{K}_S^{-1}) + \mathbf{K}_S^{-1} - \mathbf{K}_{fr}\mathbf{K}_S^{-2}},$$

Brown & Korringa:

$$\mathbf{K}_{ud} = \mathbf{K}_{fr} + \frac{(1 - \mathbf{K}_{fr}/\mathbf{K}_M)^2}{\phi(\mathbf{K}_F^{-1} - \mathbf{K}_S^{-1}) + \mathbf{K}_S^{-1} - \mathbf{K}_{fr}\boxed{\mathbf{K}_M^{-2}}}$$

Definitions of the compressibility parameters **K**:

1. K_{fr} — *Framework (or frame) incompressibility*

- This is the bulk incompressibility of the rock frame when the pores are filled with a fluid that does not support the load (i.e., a fluid with very large compressibility, like gas).
- It is sometimes equated with drained incompressibility because in both cases the pore fluid doesn't resist the applied load.

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2. K_S— *Solid (matrix) incompressibility*

- This refers to the overall incompressibility of the solid portion of the rock, which may be made up of different minerals (not necessarily homogeneous).
- It is measured under open conditions, where both internal and external pressures are equal.
- Unlike a simple average of mineral moduli, it includes microgeometry effects (e.g., how minerals are distributed).

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3. K_F— *Fluid incompressibility*

- This is the bulk modulus of the fluid that saturates the rock's pores.
- This is how compressible the pore-filling fluid is (e.g., water, oil, or gas).

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4. K_M— *Mean (volume-weighted) incompressibility*

- This is the volume weighted average of the compressibility of the fluid-filled pores and the solid:
- **K_M** thus represents the effective mean incompressibility of the rock under hydraulically closed conditions.
- The subscript "**M**" stands for mean, not matrix or mineral.